GOA UNIVERSITY
Taleigao Plateau

SYLLABUS FOR GOA UNIVERSITY ADMISSIONS RANKING TEST (GU-ART) IN MATHEMATICS

Calculus and Numerical Methods.

Real Number System: Algebra of real number system, Axioms of order structure in $\mathbb{R}$, Upper and Lower bounds of subsets of $\mathbb{R}$, lub of subsets of $\mathbb{R}$, Order completeness of $\mathbb{R}$, Archimedean property, Intervals and their types, Nested interval Theorem, Absolute value and their properties.

Real Sequences: Real Sequence (Definition and examples), Range of a sequence, Bounded sequence, Convergence of a sequence (Definition and examples), Uniqueness of limit of sequence, Algebra of sequences, Sandwich Lemma, Monotonic sequences and their convergence, Subsequence of a sequence (Definition and examples), Properties of subsequences, Bolzano Weierstrass theorem.

Limits and Continuity: Neighbourhood of a point, Deleted neighbourhood of a point, Limit of a function at a point (Definition and examples) Uniqueness of a limit, Algebra of limits, Continuity of a function at a point (Definition and examples), Algebra of continuous of function, Left hand limit, Right hand limit, Types of discontinuities, Sequential continuity, Some more properties of continuous functions, Boundedness of continuous function on a closed interval, Intermediate value theorem for continuous functions, Image of the closed interval under a continuous function, Attaining maximum and minimum of a continuous function on closed interval, Fixed point of a function, Fixed point theorem for continuous function.

Derivatives and its Applications: Derivative of a function at a point (Definition and examples), Geometric interpretation of a derivative, Algebra of derivatives, Chain rule, Some more properties of the derivative, Darboux's theorem for differentiable functions, Intermediate value theorem for differentiable functions, Rolle's theorem and its geometric significance, Lagranges mean value theorem and its geometric significance, Cauchy's mean value theorem and its geometric significance, Monotonic functions (Definition and examples), Monotonic functions and derivatives, Higher order derivatives, Taylor's theorem, Mclaurin's theorem, Taylor's and Mclaurin's series expansions, Leibnitz rule for higher order derivative of product of functions, Stationary points and their classification, Local maxima and Local minima, Condition for a stationary point to be local maxima and minima, Indeterminate forms of the type $\frac{0}{0}$, $\frac{\infty}{\infty}$, $\infty - \infty$, $0.\infty$, 0^0, 1^∞, ∞^0.

Numerical Methods:

Calculus of Finite differences: Operators Δ, ∇, & E . Difference Tables. Properties of Δ, ∇, & E. Fundamental Theorem of Difference Calculus. Expression of any value of a function in terms of leading term and leading differences of a difference table. Method of separation of symbols.

Interpolation and Extrapolation: Newton's forward and backward interpolation formulae. Lagrange's Interpolation formula . Newton's Divided Difference formula.

Examples based on the above formulae.

Numerical Differentiation and Integration: Differentiation formulae for equidistant arguments. Examples. General quadrature formula for equidistant ordinates (Newton –Cotes Formula Or Gauss Legendre quadrature formulae). Trapezoidal rule and its Geometrical interpretation. Simpson's one third rule, Simpson's three-eighth rule. Weddle's rule (Only Statements).

Solution of Algebraic and transcendental Equations: Method of Bisection, Regula-Falsi Method, Newton-Raphson Method and their Computation scheme. Special Cases of Newton-Raphson Method like finding q^{th} root of a positive real number'd' and finding reciprocal of a positive real number 'd' without using division.

Matrices and Linera Algebra.

System of linear equations [Nicholson, Chapter 1]: Solutions & Elementary Operations: (Linear system of equations, solutions, equivalence of 2 systems, elementary operations on equations, elementary row operations). Gaussian Elimination: (Row /Row reduced echelon forms, Gaussian algorithm, Rank). Homogeneous Equations: (Sufficient condition for the existence of non-trivial solution)

Matrix Algebra[Nicholson, Chapter 2]: Matrix Addition, Scalar multiplication, Transposition: (Definition, properties, symmetric matrix). Matrix Multiplication:(Definition, properties, block multiplication). Matrix Inverses: (Definition, uniqueness, properties, Matrix inversion algorithm(row reduction)). Elementary Matrices: (Definition, properties, theorems).

Determinants [Nicholson, Chapter 3]: The Laplace Expansion: (Determinant, properties, upper/ lower triangular matrices). Determinant & Matrix inverses (Product theorem & other related theorems, orthogonal matrices, minors, co-factors, adjoint formula for A^{-1} , Cramer's rule)

Vector Spaces: Definition and examples, Vector subspaces, Basis and Dimension of Vector Spaces.

Lines and Quotient Spaces: Definition of a line, Affine spaces, Quotient Spaces.

Linear Transformations: Linear Transformation, Representation of linear maps by matrices, Kernel and Image of a Linear Transformation, Linear Isomorphism, Geometric ideas and some loose ends, Some special Linear Transformations.

Inner Product Spaces: Inner Product Spaces, The Euclidean plane and the dot product, General Inner Product Spaces, Orthogonality, Some geometric applications, Orthogonal projection onto a line, Orthonormal basis, Orthogonal complements and projections, Linear Functionals and Hyperplanes, Orthogonal Transformations, Coordinates associated with an Orthonormal Basis, Reflections and Orthogonal Maps of the Plane.

Diagonalization: Rotation of axes of conic, Eigenvalues and eigenvectors, Cayley-Hamilton theorem, Diagonalisation of symmetric matrices.

Review Problems: Linear equations, Linear dependence, Basis and Dimension, Linear Transformations, Euclidean Spaces, Problems in Linear Geometry, Miscellaneous problems.

ORDINARY DIFFERENTIAL EQUATIONS:-

Review of Basic concepts such as order, degree, formation, solution and their types of differential equations. First order first degree differential equation and initial value problem. Method of separation of variables. Homogeneous and Non - homogeneous differential equation. First order linear differential equations. Bernoulli's differential equation. Exact and Non – exact differential equations. Condition for exactness. Integrating factors and rules to find integrating factors. Clairaut's differential equation and differential equations reducible to Clairaut's form. Ricatti's differential equation. Applications of first order differential equations. Modeling with differential equations. General form of second order linear differential equation and its classification. Existence and Uniqueness theorem for solution of second order linear differential (Only statement). Second order homogeneous linear differential equation and its properties. Wronskian of solutions of homogeneous linear differential equation and its properties. Linear dependence and independence of solutions of homogeneous differential equation. Complementary function. Use of known solution to find second linearly independent solution of homogeneous differential equation. Homogeneous linear differential equations with constant coefficients and with variable coefficients. Method of undetermined coefficients. Method of variation of parameters. Applications of second order linear differential equations.

D – Operator method to solve n^{th} order homogeneous linear differential equation with constant coefficients. Properties of D – Operator. Inverse D – operator and it properties. Inverse D – operator method to solve n^{th} order Non - homogeneous linear differential equation with constant coefficients **f(D) = R(x)**, where **R(x) = e^{ax} , cos ax, sin ax**, polynomial in '**x**' and their products.

Conversion of n^{th} order differential equation to first order system of differential equations. Existence and uniqueness of solution (statement only). " 2×2 " homogeneous linear first order system of differential equations and their solution. Wronskian of " 2×2 " homogeneous linear first order system of differential equations and its properties. Linear dependence and independence of solutions of " 2×2 " homogeneous linear first order system of differential equations. Matrix method to solve " 2×2 " homogeneous linear first order system of differential equations with constant coefficients. Method of solving 2×2 Non - homogeneous linear first order system of differential equations with constant coefficients.

GRAPH THEORY:-

Introduction. Basic terminology. Types of Graphs. Multigraphs and Weighted graphs. Isomorphism of graphs. Paths and circuits. Shortest path in weighted graphs. Eulerian paths and circuits. Hamiltonian paths and circuits. Factors of graphs. planar graphs. Trees. Rooted trees. Path lengths in rooted trees. Prefix codes. Binary search trees. Spanning trees and cut- sets. Minimum spanning tree. Kruskal's algorithm. Prim's algorithm. Transport network.

Analysis

Convergence of infnite series, absolute convergence, Conditional convergence, Geometric series, Cauchy criterion for convergence, Algebra of convergent series, Comparision test, Convergence of Harmonic P-series, D'Alembert ratio test,Cauchy nth root test,Leibniz test or alternating series test**.**

Darboux Integrability, Criterion for integrability, Properties of integrabilities. First fundamental theorem of calculus, Second fundamental theorem of calculus, integration by parts, Mean value theorems for integrals, First mean value theorem for integrals, Second mean value theorem I, Second mean value theorem II, Riemann original definition**.**

Pointwise convergence of sequence of functions and examples, Uniform convergence of sequence of functions and examples, Mn-Test,Cauchy Criterion for uniform convergence, Consequences of Uniform convergence, Continuity of limit function, Series of functions, Absolute convergence, Cauchy Criterion for uniform convergence of a series, Weierstrass M-test, Weierstrass Approximation Theorem.

Operations Research :

Fundamentals: Linear Programming problems, Convex sets ,Extreme points of Convex sets, Convex Polyhedron ,hyperplanes, Graphical Method, Simplex Method, Theorems on simplex method ,Big-M method, Two phase method, Unrestricted variables, Duality and solution using duality, Theorems on Duality, Dual Simplex method, Post Optimal Analysis (Discrete changes in cost and requirement vector) Transportation Problems, North west corner method, Vogel's approximation method, Modi Method, Assignment Problems, Hungrian Method, Basics of Inventory control, Inventory model with No shortages and Instantaneous production, Inventory model with Shortages allowed and Instantaneous production. Basics of Queueing theory, Queueing Model (M/M/1):(1/FIFO),
Queueing Model (M/M/1):(N/FIFO).

References:

1. A Basic Course in Real Analysis, Ajit Kumar & S. Kumaresan, CRC Press.
2. Introductory Methods of *Numerical Analysis* By S.S.Sastry. PHI
3. Linear algebra with applications by Keith Nicholson (3rd Edition, PWS publishing company).
4. Linear Algebra, A Geometric Approach, S. Kumaresan, PHI Learning Private Limited. (Chapter 6 omitted.)
5. Differential Equations and Their Applications: Martin Braun (Springer)
6. Ordinary Differential Equations: E. A. Coddington (PHI Learning Pvt. Ltd.)
7. Differential Equations by G.F. Simmons, TMH.
6. Elements of Discrete Mathematics: C. L. Liu and D. P. Mohapatra (Tata Mcgraw Hill)
7. Linear Programming by G. Hadley; Adddison.
8. Operations Research, Kanti Swarup and Gupta, S. Chand and company , Delhi.